COMBINATION OF TWO OPTIMIZATION TECHNIQUES

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For global optimization, an algorithm based on electromagnetism theory in physics (EM method) has been proposed, using an attraction-repulsion mechanism to move the sample points towards the optimality. Another method for optimization, called neuroevolution (evolving neural network), is developed. It utilizes a learning mechanism and is based on the neural network, which is motivated by biological structure of neurons.
In this talk, I will first introduce two methods above. Then I will propose a combination of EM method and evolving neural network, or give some plans on combining two.

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ON OPTIMALITY CONDITIONS IN CONVEX OPTIMIZATION WITH LOCALLY LIPSCHITZ CONSTRAINS

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In this talk, we consider a convex optimization problem with locally Lipschitz inequality constraints. The KKT optimality conditions for quasi ϵ-solutions are established under Slater's constraint qualification and a non-degeneracy condition. Moreover, we explore the optimality condition for weakly efficient solutions in multiobjective convex optimization involving locally Lipschitz constraints.

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DEFINING EQUATIONS OF RATIONAL CURVES IN SMOOTH QUADRIC SURFACE

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For a nondegenerated irreducible projective variety, it is a classical but very difficult problem to study the defining equations of a variety with respect to the given embedding. In general, the varieties of which defining equations are completely known are very rare. Let $C \subset \mathbb{P}^3$ be a rational curve of degeree $d \geq 3$ which is defined to be an image of the map $\nu_d : \mathbb{P}^1 \longrightarrow \mathbb{P}^3$ parameterized by

$$C_d = \{[s^d(P) : s^{d-1}t(P) : st^{d-1}(P) : t^d(P)] \mid P \in \mathbb{P}^1\}.$$

In this talk, we precisely determine the defining equations of rational curves $C \subset \mathbb{P}^3$ for all $d \geq 3$

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NEURAL NETWORKS BASED ON THREE CLASSES OF NCP-FUNCTIONS FOR SOLVING NONLINEAR COMPLEMENTARITY PROBLEMS

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We consider a family of neural networks for solving nonlinear complementarity problems (NCP). The neural networks are based from the merit functions induced by three classes of NCP-functions: the generalized natural residual function and its two symmetrizations. We first provide a characterization of the stationary points of the induced merit functions. To prove the boundedness of the level sets of the merit functions, we prove some important properties related to the growth behavior of the complementarity functions. Furthermore, we analyze the stability of the steepest descent-based neural network model for NCP. To illustrate the theoretical results, we provide numerical simulations using our neural network and compare it with other similar neural networks in the literature which are based on other well-known NCP-functions. The numerical results suggest that the neural network has a better performance when their common parameter p is smaller. We also found that one among the three families of neural networks we considered is capable of outperforming other existing neural networks.

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A GAUGE-INVARIANT HIGH-ORDER MIXED FEM SCHEME FOR TDGL EQUATIONS ON 2D NON-CONVEX DOMAINS

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The microscopic description of the vortex state for the type-II superconductors can be described by the time-dependent Ginzburg-Landau (TDGL) equations, which are given by

$$\eta\frac{\partial\Psi}{\partial t} + i\eta\kappa\Phi\Psi + \left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)^2\Psi + (|\Psi|^2 - 1)\Psi = 0, \text{ on } \Omega\times(0,T],$$

$$\frac{\partial\mathbf{A}}{\partial t} + \nabla\Phi + \mathrm{Re}\left[\Psi^*\left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)\Psi\right] + \nabla\times\nabla\times\mathbf{A} - \nabla\times\mathbf{H} = 0,$$

with the boundary and initial conditions

$$\left(\frac{i}{\kappa}\nabla + \mathbf{A}\right)\Psi\cdot\mathbf{n} = 0, \ \ \nabla\times\mathbf{A} = \mathbf{H}, \text{ on } \partial\Omega\times[0,T],$$

$$\Psi(x,0) = \Psi_0(x), \ \ \mathbf{A}(x,0) = \mathbf{A}_0(x), \text{ on } \Omega,$$

where $\Omega \subset \mathbf{R}^2$ is the region occupied by the superconducting sample and $\mathbf{n}$ is the unit outer normal vector on the boundary $\partial\Omega$. The variable functions Ψ is the complex scalar-valued order parameter, $\mathbf{A}$ is the real vector-valued magnetic potential, and Φ is the real scalar-valued electric potential. Physically, $|\Psi| = 0$ and $|\Psi| = 1$ correspond to the normal state and the superconducting state respectively, while $0 < |\Psi| < 1$ represents a mixed (vortex) state. The real vector-valued function $\mathbf{H}$ is the applied magnetic filed, κ is the Ginzburg-Landau parameter, and η is the normalized conductivity (usually set as 1 for simplicity).

In this talk, I will sketch the background of TDGL equations and introduce the equations in detail, especially the gauge-invariant property. Then I will survey some numerical methods published in the past two decades and give the motivation of the high-order mixed finite element method (FEM) on non-convex domains mentioned in the title. Finally, I will discuss my current progress and the ongoing work.

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LOCAL STABILITY AND LOCAL CONVERGENCE OF THE BASIC TRUST-REGION METHOD

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It is about the Basic Trust-Region Method for solving the nonlinear optimization problems. We proved that the iterative sequence constructed by this algorithm, which uses the Cauchy point method, is locally stable and linearly convergent in a neighborhood of a nonsingular local minimizer.

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THE THEORY OF FALLING SHADOWS APPLIED TO B-ALGEBRA

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J. Neggers and H. S. Kim introduced the notion of B-algebra in 2002 [5]. B-algebra is related to several classes of algebras of current interests such as $BCH/BCI/BCK$-algebras and is described as follows: A B-algebra is a nonempty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms: (B1) $x * x = 0$; (B2) $x * 0 = x$; (B3) $(x * y) * z = x * (z * (0 * y))$, for all $x, y, z \in X$.

In this paper, the notion of falling fuzzy ideal of B-algebras which is based on the theory of falling shadows ([1, 6]) is introduced. Several properties about falling fuzzy ideal are also investigated and established. Consequently, the concept of t-norm T ([2, 3, 4]) is applied to fuzzy ideal structure of B-algebras from which the T-fuzzy ideal of B-algebras is proposed. Some characterizations of T-fuzzy ideals are also presented and conditions for a falling fuzzy ideal to be a T-fuzzy ideal are provided.

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ON A GRAPH INDUCED BY A HYPER BCI-ALGEBRA

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The study of BCI-algebra was initiated by K. Iséki in 1966 [1]. Various studies has been developed to establish some of its properties and further investigations yield to elevate its structure into its hyperstructure. Let H be a nonempty set and $\circledast$ a function from $H \times H$ to $P^*(H)$, where $P^*(H)$ denotes the power set of $H \setminus \{\varnothing\}$. Then we call $(H, \circledast)$ a *hyper groupoid* and $\circledast$ a *hyperoperation*. By a *hyper BCI-algebra* H, we mean a hyper groupoid $(H, \circledast)$ that contains a constant 0 and satisfies the following axioms: $((x \circledast z) \circledast (y \circledast z)) \ll x \circledast y$; $(x \circledast y) \circledast z = (x \circledast z) \circledast y$; $x \ll x$; $x \ll y$ and $y \ll x$ implies $x = y$; and $0 \circledast (0 \circledast x) \ll x$, $x \neq 0$, for all $x, y, z \in H$. The concept of a hyper BCI-algebra which is a generalization of a BCI-algebra was introduced by X.L. Xin [2]. Since then, various properties of this algebraic hyperstructure have been studied by several authors. This paper introduces the notion of the zero divisor graph of a hyper BCI-algebra and investigates some of its properties.

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APPROACH TO THE SOLUTION EXISTENCE PROOF FOR NAVIER–STOKES EQUATION BY USING VERIFIED COMPUTING

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As one of the Millennium Prize Problems, the problem of existence and smoothness of the Navier–Stokes equation problem draws attention of mathematicians from the world. To solve the this problem, a breakthrough with new mathematical theory and new computing technique is expected. On one hand, the computer-assisted proof utilizing verified computing, which aims to give rigorous estimation for all error appearing in numerical computations, provides a new approach to the solution existence proof to the Navier–Stokes equation. In this talk, I will explain the basic idea of verified computing and show the latest progress in this field, which includes recent research on rigorous error estimation for finite element method solution to Navier-Stokes equations.

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RELATIONSHIPS BETWEEN CONSTRAINED AND UNCONSTRAINED MULTI-OBJECTIVE OPTIMIZATION AND APPLICATION IN LOCATION THEORY

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(JOINT WORK WITH CHRISTIAN GÜNTHER)

The main purpose of the lecture is to investigate relationships between constrained and unconstrained multi-objective optimization problems. We mainly focus on generalized convex multi-objective optimization problems, i.e., the objective function is a componentwise generalized convex (e.g., quasi-convex, semi-strictly quasi-convex or explicitly quasi-convex) function and the feasible domain is a convex set. However, it is important to note that we derive several results for general objective functions without generalized convexity assumptions on the objective function. Beside the field of location theory the assumptions of generalized convexity are found in several branches of Economics, e.g., in the field of utility theory.

In the lecture, we formulate the basic constrained multi-objective optimization problem and the corresponding extended unconstrained one, we introduce solution concepts and recall results about generalized convex and semi-continuous functions. Moreover, we introduce gauge functions and we prove some important facts about gauges. These results will be used for deriving characterizations of the sets of solutions of constrained multi-objective optimization problems.

Under suitable assumptions (e.g., generalized convexity assumptions) we derive a characterization of the set of (strictly, weakly) efficient solutions of a constrained multi-objective optimization problem using characterizations of the set of (strictly, weakly) efficient points of unconstrained multi-objective optimization problems. Furthermore, we present a theorem that provides lower and upper bounds for the sets of (strictly, weakly) efficient solutions for multi-objective optimization problems involving nonconvex constraints.

We apply our results to constrained point-objective location problems involving mixed gauges defined by

$$\begin{pmatrix} \eta_1(x-a^1) \\ \dots \\ \eta_m(x-a^m) \end{pmatrix} \to \min_{x\in X},$$

where $\eta_1,\dots,\eta_m : \mathbb{R}^n \to \mathbb{R}$ represent special distance functions (gauges) and $a^1,\dots,a^m$ are finitely many given points in $\mathbb{R}^n$. We present several examples in order to illustrate that the sets of (strictly, weakly) efficient solutions can be completely generated for convex constrained point-objective location problems using known algorithms for the unconstrained case taking into account our results.

Finally, we demonstrate the MATLAB-based software tool

Facility **L**ocation **O**ptimizer (**FLO**)

that can be used for solving special types of single- as well as multi-objective location problems involving different distances measures. For more information, see `http://www.project-flo.de`.

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ON APPROXIMATE SOLUTIONS FOR ROBUST CONVEX OPTIMIZATION PROBLEMS

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Mathematical optimization problems in the face of data uncertainty have been treated by the worst case approach or the stochastic approach. The worst case approach for optimization problems, which has emerged as a powerful deterministic approach for studying optimization problems with data uncertainty, associates an uncertain optimization problem with its robust counterpart.

Many researchers have investigated optimality and duality theories for linear or convex programming problems under uncertainty with the worst-case approach(the robust approach). Moreover, many authors have studied optimality and duality theorems for robust multiobjective optimization problems under different suitable constraint qualifications.

In this talk, we explain robust optimization problems with an illustrating example, and then we review our recent results on optimality theorems and duality theorems for robust convex optimization problems.

(1) In the first section, we explain "What is a robust optimization problem?";

(2) In the second section, we consider approximate solutions(ϵ-solutions) for a robust convex semidefinite optimization problem involving a convex objective function and linear matrix constraint functions with data uncertainty. Using robust optimization approach(worst-case approach), an approximate optimality theorem and approximate duality theorems for the problem are given;

(3) In the third section, we consider approximate solutions(ϵ-solutions for a robust convex semi-infinite optimization problem involving a convex objective function and infinitely many convex constraint functions with data uncertainty, and give its robust counterpart. Using robust optimization approach (worst-case approach), approximate optimality theorem and approximate duality theorems for the problem are given.

The second section is based upon the paper "On Approximate Solutions for Robust Convex Semidefinite Optimization Problem" which was published in the Journal "Positivity" (2018) and which was written by Jae Hyoung Lee and myself, and the third section is based upon the paper "On ϵ-Solutionss for Robust Semi-infinite Optimization Problems" which will be published in the Journal "Positivity" and which was written by Jae Hyoung Lee and myself.

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AN INERTIAL FORWARD-BACKWARD SPLITTING ALGORITHM FOR MONOTONE INCLUSIONS WITH APPLICATIONS

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The purpose of this paper is to introduce a new iterative method that is the combination of the modified Mann type forward-backward splitting, viscosity approximation method and alternating resolvent method for finding the zero of sum of accretive operators in uniformly convex real Banach spaces which are also uniformly smooth algorithm. Our result is new and complements many recent and important results in this direction in the literature.

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INCREMENTAL TYPE METHODS FOR ADDITIVE CONVEX MINIMIZATION PROBLEMS

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We will start by focusing on some important examples, from a variety of applications, which arise in several important contexts: an objective function is the sum of a large number of components,

$$f(x) := \sum_{i=1}^{m} f_i(x), \tag{1}$$

where the functions f_i are convex real-valued. The problem of type (1) is called *additive cost problem.*

The additive cost problem can be minimized with specialized methods, called incremental, which exploit their additive structure, by updating x using one component function f_i at a time. Subsequently, in this talk, we will discuss the ideas underlying incremental method and its convergence properties. Finally, the interesting combinations of the incremental concept with other methods, such as the proximal algorithm will be considered.

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STRONG DUALITY IN MINIMIZING A QUADRATIC FORM SUBJECT TO TWO HOMOGENEOUS QUADRATIC INEQUALITIES OVER THE UNIT SPHERE

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This problem, called (P), is a generalization of a simpler version (P') which also minimizes a quadratic form but has just one homogeneous quadratic constraint over the unit sphere. The inclusion of an additional homogeneous quadratic constraint can cause (P) to have a positive duality gap whereas the simpler version (P') has been proved to adopt strong duality under Slater's condition. On the surface the underlined problem (P) appears to be different from the CDT (Celis-Dennis-Tapia) problem. Their SDP relaxations, however, share a very similar format. The minute observation turns out to be valuable in deriving a necessary and sufficient condition for (P) to admit strong duality. We will see that, in the sense of strong duality results, problem (P) can be also considered as a generalization of the CDT problem and even more, we can solve (P) without assuming the Slater's condition. Many nontrivial examples are to be shown in the talk to help understand the mechanism.

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